

### Systems of Equations (Substitution)

**Aim:** How do we use substitution to solve a system of linear equations?

- The substitution method is used to solve systems of linear equations by solving an equation for one variable and then substituting the resulting expression for that variable into the other equation.

**Example 1:** Solve the system graphically:

$$y = 2x - 3 \quad y = -2x + 5$$

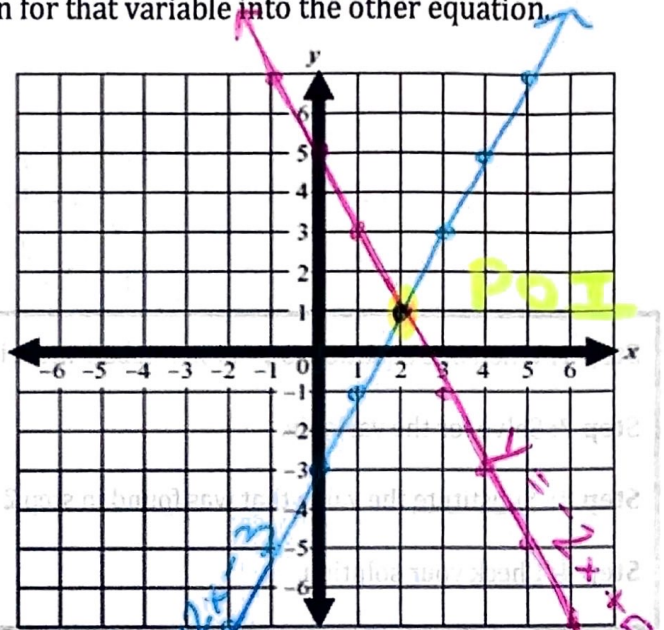
$$m = \frac{2}{1}$$

$$m = \frac{-2}{1}$$

$$b = (0, -3)$$

$$b = (0, 5)$$

**Solution:**  $(2, 1)$



Solve the same system from above, **algebraically**:

$$y = 2x - 3$$

$$y = -2x + 5$$

★ Both equal to y  
so they are equal

to each other! ★

**VOBS**

$$\begin{array}{r} 2x - 3 = -2x + 5 \\ +2x \quad \quad +2x \\ \hline 4x - 3 = 5 \\ +3 \quad \quad +3 \\ \hline 4x = 8 \\ \frac{4x}{4} = \frac{8}{4} \end{array}$$

- Step 1:** When both equations are  $y =$  or  $x =$ , set them equal to each other.
- Step 2:** Solve for the variable.
- Step 3:** Substitute the value that was found in step 2 into either equation to find the value of the other variable.
- Step 4:** Check your solution.  
(2 equations means 2 checks)

★ substitute  $x=2$  into either equation ★

$$y = 2x - 3$$

$$y = 2(2) - 3$$

$$y = 1$$

check

$$y = 2x - 3$$

$$1 = 2(2) - 3$$

$$1 = 1 \checkmark$$

$$x = 2$$

check

$$y = -2x + 5$$

$$1 = -2(2) + 5$$

$$1 = 1 \checkmark$$

**Solution:**  $(2, 1)$

Hmmm... Did we get the **same solution** from graphing as we did using the substitution method?

Try it: Solve the system algebraically.

$$\begin{array}{r}
 x + 2 = 3x - 4 \\
 -x \quad -x \\
 \hline
 2 = 2x - 4 \\
 +4 \quad +4 \\
 \hline
 6 = 2x \\
 \frac{6}{2} = \frac{2x}{2} \\
 3 = x
 \end{array}$$

$$\begin{array}{l}
 y = x + 2 \\
 y = 3 + 2 \\
 \boxed{y = 5}
 \end{array}$$

solution  
(3, 5)

$$\begin{array}{l}
 y = x + 2 \\
 y = 3x - 4
 \end{array}$$

Check

$$\begin{array}{ll}
 y = x + 2 & y = 3x - 4 \\
 5 = 3 + 2 & 5 = 3(3) - 4 \\
 5 = 5 \checkmark & 5 = 5 \checkmark
 \end{array}$$

Step 1: When one equation is  $y =$  or  $x =$ , substitute it into the other equation.

Step 2: Solve for the variable.

Step 3: Substitute the value that was found in step 2 into either equation to find the value of the other variable.

Step 4: Check your solution.

Example 2: Solve the system algebraically.

a)  $y = 5$   
 $3x + 2y = 25$

$$\begin{array}{r}
 3x + 2(5) = 25 \\
 3x + 10 = 25 \\
 -10 \quad -10 \\
 \hline
 3x = 15 \\
 \frac{3x}{3} = \frac{15}{3} \\
 \boxed{x = 5}
 \end{array}$$

Check

$$\begin{array}{l}
 3x + 2y = 25 \\
 3(5) + 2(5) = 25 \\
 15 + 10 = 25 \\
 25 = 25 \checkmark
 \end{array}$$

\* plug in to 1st eq. b/c easier  
 $y = x - 3$   
 $y = 5 - 3$   
 $\boxed{y = 2}$

★ Don't need to solve for  $y$  because it was given  $\boxed{y = 5}$

Solution:  $\underline{(5, 5)}$

b)  $y = x - 3$   
 $2x + 3y = 16$

$$\begin{array}{r}
 2x + 3(x - 3) = 16 \\
 2x + 3x - 9 = 16 \\
 5x - 9 = 16 \\
 +9 \quad +9 \\
 \hline
 5x = 25 \\
 \frac{5x}{5} = \frac{25}{5} \\
 \boxed{x = 5}
 \end{array}$$

Check

$$\begin{array}{l}
 y = x - 3 \\
 2 = 5 - 3 \\
 2 = 2 \checkmark
 \end{array}$$

$$\begin{array}{l}
 2x + 3y = 16 \\
 2(5) + 3(2) = 16 \\
 10 + 6 = 16 \\
 16 = 16 \checkmark
 \end{array}$$

Solution:  $\underline{(5, 2)}$



Try It: Solve the system algebraically.

$$5(2-y) - 2y = 3$$

$$10 - 5y - 2y = 3$$

$$10 - 7y = 3$$

$$-10 \quad -10$$


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$$-7y = -7$$

$$-\cancel{7}y = \cancel{-7}$$

$$y = 1$$

Find  $x$

$$x = 2 - y$$

$$x = 2 - 1$$

$$x = 1$$

Check

$$x = 2 - y$$

$$1 = 2 - 1$$

$$1 = 1 \checkmark$$

$$5x - 2y = 3$$

$$5(1) - 2(1) = 3$$

$$5 - 2 = 3$$

$$3 = 3 \checkmark$$

**Step 1:** Pick one equation to solve for one of the variables. (Get one equation to look like  $x =$  or  $y =$ )

**Step 2:** Substitute the expression into the variable in the other equation.

**Step 3:** Solve for variable.

**Step 4:** Substitute the value that was found in Step 3 into the other equation that wasn't used to find the other variable.

**Step 5:** Check your solution.

Example 3: Solve the system algebraically.

$$x - 2y = 5$$

$$+ 2y = +2y$$

$$x = 2y + 5$$

$$3x - 5y = 8$$

Find  $x \Rightarrow$  in either eq.

$$x - 2y = 5$$

$$x - 2(-7) = 5$$

$$x + 14 = 5$$

$$-14 \quad -14$$

$$x = -9$$

$$3(2y + 5) - 5y = 8$$

$$6y + 15 - 5y = 8$$

$$y + 15 = 8$$

$$-15 \quad -15$$

$$y = -7$$

$$x - 2y = 5$$

$$3x - 5y = 8$$

check

$$x - 2y = 5$$

$$-9 - 2(-7) = 5$$

$$-9 + 14 = 5$$

$$5 = 5 \checkmark$$

$$3x - 5y = 8$$

$$3(-9) - 5(-7) = 8$$

$$-27 + 35 = 8$$

$$8 = 8 \checkmark$$

Solution:  $(-9, -7)$